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PHYSICS:

ELECTRICITY AND MAGNETISM:

Laboratory works

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as a study aid for the foreign students
for the specialties 131 Applied mechanics; 133 Manufacturing engineering
134 Aviation, rocket and space machinery; 173 Avionics
of the Institute of Mechanical Engineering

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ELECTRICITY AND MAGNETISM:
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ANNOTATION
For the educational publication
“Physics: Electricity and magnetism: Laboratory works”

Methodical recommendations for laboratory works in Physics for students who study section "Electricity and magnetism" in physics and are under the Bachelor’s degree study program the for the specialties 131 Applied mechanics; 133 Manufacturing engineering; 134 Aviation, rocket and space machinery; 173 Avionics of the Institute of Mechanical Engineering. It also could be used for other students’ specialties at the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”.

Methodical recommendations for laboratory works in Physics are designed for the foreign students and written in English, they are understandable and at the same time they correspond to the "Physics" course curriculum for the Institute of Mechanical Engineering by the level of material presentation. Methodical recommendations are a practical guide for performing laboratory works in the laboratories of the Faculty of Physics and Mathematics. They provide students with an opportunity to get acquainted with fundamental laws of physics and to verify directly the implementation of these laws in experiments, to form a sufficient level of competence for carrying out physical experiments, processing data and estimating results.

There are laboratory works from the section "Electricity and magnetism" in the present publication, namely, such topics as "Definition of electrical resistance with a Wheatstone bridge", "Definition of the electromotive force using the method of compensation", "Determination of capacitance using a ballistic galvanometer", "Study of the electrostatic field".

The text of the protocol of each laboratory work is accompanied by necessary explanations, illustrations, tables, description of the experimental setup, the procedure order and processing of the experimental results, control questions.

Laboratory work № 2.1

DEFINITION OF ELECTRICAL RESISTANCE WITH A WHEATSTONE BRIDGE

Objective: to study direct current laws and master classical method of measuring resistance using a bridge circuit

Equipment: battery of known emf 6V; galvanometer; variable resistor up to 1000 Ohm; studied resistors; switch and connecting wires.

Theoretical information

A Wheatstone bridge is a classical electrical circuit used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component.

Calculation of such circuits is simplified using two Kirchhoff's current laws:

1. Junction rule (also called as Kirchhoff's first law or Kirchhoff's current law)

   At any junction (node), the sum of the currents must equal zero: the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

$$\sum_{i=1}^{n} I_i = 0 \qquad (1.1)$$

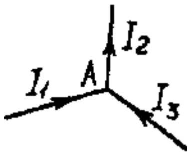
   Junction (node) is a point at which more than two conductors are connected (Fig. 1.1). The current flowing into the node is considered to have positive sign, while the current flowing out of the node is of negative sign.

   Similar equations can be written for each of the *N* nodes of the electric circuit. Thus, we obtain (*N* -1) independent equations, and the *N*-th equation follows from them.

2. Loop rule (also called as Kirchhoff's second law or Kirchhoff's voltage law).

   The sum of the emfs in any closed loop is equivalent to the sum of the potential drops in that loop.

$$\sum_{i=1}^{n} I_i R_i = \sum_{i=1}^{n} \varepsilon_i \qquad (1.2)$$



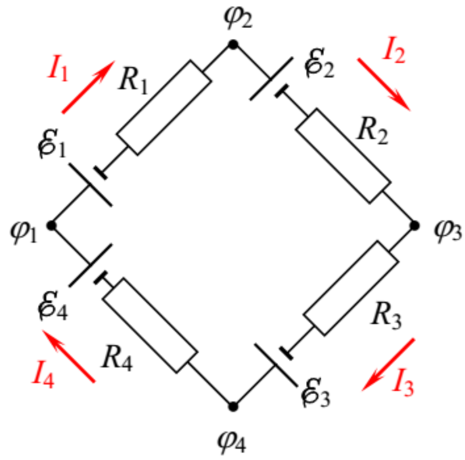


Figure 1.1 Figure 1.2

The potential drop is the product of the resistance of the conductor and the current trough it: $I_i R_i$.

To apply the Kirchhoff's second law, first we have to specify positive direction of traveling around the loop: clockwise or counterclockwise. If direction of the current I_i coincides with the travelling direction, the sign of the potential drop $I_i R_i$ is positive, otherwise it is negative. Similarly, if direction of the emf ε_i coincides with the travelling direction (i.e. if the emf heightens electric potential in the travelling direction) the sign of the emf is positive, otherwise it is negative. The potential energy increases whenever the charge passes through the emf from the negative terminal to the positive terminal.

The Kirchhoff's second law can be derived using the Ohm's law for each of the branches of the circuit (1, 2, 3, 4), shown in Fig. 1.2.:

$$\begin{aligned} I_1 R_1 &= (\varphi_1 - \varphi_2) - \varepsilon_1 \\ I_2 R_2 &= (\varphi_2 - \varphi_3) - \varepsilon_2 \\ I_3 R_3 &= (\varphi_3 - \varphi_4) + \varepsilon_3 \\ I_4 R_4 &= (\varphi_4 - \varphi_1) + \varepsilon_4 \end{aligned} \tag{1.3}$$

where I_1, I_2, I_3, I_4, are currents; R_1, R_2, R_3, R_4 are resistances of the corresponding circuit branches, $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ are the potentials of the corresponding points; $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ are emf.

In the result of adding the equations of the system (1.3) we obtain the Kirchhoff's second law (1.2).

The basic diagram of the Wheatstone bridge circuit is depicted in Fig. 1.3. In the figure, R_x is the unknown resistance to be measured; R_2, R_3 are resistors of known resistance and the resistance of the variable resistor R_1 is adjustable.

Each of the four resistances in a bridge circuit are referred to as "legs". The resistor in series with the unknown resistance R_x (this would be R_1 in the Fig. 1.3) is commonly called the rheostat of the bridge, while the other two resistors are called the ratio legs of the bridge.

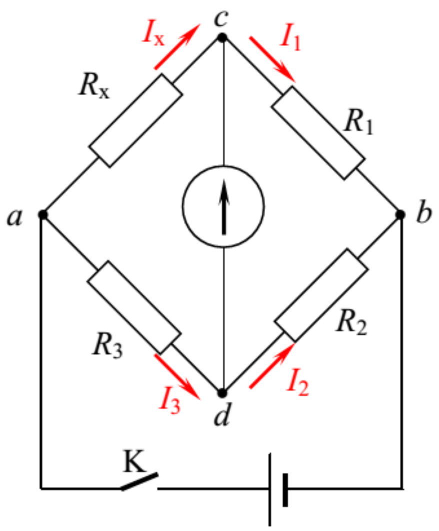


Figure 1.3.

Applying the Kirchhoff's second law to the loops *acd* and *cbd*, we obtain (positive travelling direction is clockwise):

$$\begin{cases} I_x R_x + I_r R_r - I_3 R_3 = 0, \\ I_1 R_1 - I_2 R_2 - I_r R_r = 0, \end{cases} \tag{1.4}$$

where I_r is current through the bridge branch *cd*, R_r is its resistance.

For nodes *c* and *d*, according to Kirchhoff's first law, we have:

$$\begin{cases} I_x - I_r - I_1 = 0, \\ I_r + I_3 - I_2 = 0, \end{cases} \tag{1.5}$$

The resistance R_1 is adjusted until the bridge is "balanced" and no current flows through the galvanometer ($I_r = 0$). Then, from (1.4) and (1.5) we obtain:

$$\begin{cases} I_x R_x - I_3 R_3 = 0, \\ I_1 R_1 - I_2 R_2 = 0, \end{cases} \tag{1.6}$$

$$\begin{cases} I_x = I_1, \\ I_3 = I_2. \end{cases} \tag{1.7}$$

Substituting (1.7) into (1.6),

$$I_1 R_x = I_3 R_3,$$
$$I_1 R_1 = I_3 R_2$$

and then dividing first equation by second equation, we obtain

$$R_x = R_1 \frac{R_3}{R_2}. \tag{1.8}$$

The described method allows us to determine the unknown resistance R_x, knowing the resistances R_1, R_2, R_3.

The bridge method has limitations. It can not be used to measure small resistances, since under such conditions a large current will flow through the circuit, resulting in significant measurement errors.

Procedure

The task is to define values of five unknown resistances: R_{1x}, R_{2x}, R_{3x}, R_{4x}, R_{5x}.

Definition of resistance R_{1x}.

1. Turn on the DC power supply (± 6 V).
2. Set the arbitrary (or maximum) value of the resistance R_1 of the rheostat leg of the bridge and briefly close the switch K. Look at the arrow of the galvanometer. Vary the resistance R_1 until the galvanometer reading becomes zero.
3. Write this value of R_1 and find the unknown resistance R_{1x} using the formula:

$$R_x = R_1 \frac{R_3}{R_2}.$$

Resistances R_3 and R_2 are given.

4. Repeat steps 2-3 for four more times, unbalancing the bridge before each measurement and balancing it again by finding the value of the resistance R_1 at which no current flows through the galvanometer. Fill the table 1.1 with the obtained results. Thus, n = 5 measurements are made (i = 1,2,3,4,5), that is, we obtain five independent values of resistance R_{1x}.
5. According to tab. 1.1 calculate the average value of $\langle R_{1x} \rangle$:

$$\langle R_{1x}\rangle = \frac{1}{n}\sum_{i=1}^{n} R_{1xi}$$

6. Repeat the same measurements and calculations for each of the unknown resistances $R_{2x}, R_{3x}, R_{4x}, R_{5x}$.

Table 1.1

i	R_{1xi}	R_{2xi}	R_{3xi}	R_{4xi}	R_{5xi}
1					
2					
3					
4					
5					
	$\langle R_{1x}\rangle$=……..	$\langle R_{2x}\rangle$=……..	$\langle R_{3x}\rangle$=……..	$\langle R_{4x}\rangle$=……..	$\langle R_{5x}\rangle$=……..

7. Estimate the sample standard deviation of the mean for one of the resistances, for example, R_{1x}:

$$S_{\langle R_{1x}\rangle} = \sqrt{\frac{\sum_{i=1}^{n}(R_{1xi}-\langle R_{1x}\rangle)^2}{n(n-1)}} =$$

8. Estimate the systematic error for one of the resistances $\sigma_{\langle R_{1x}\rangle}$,

$$\sigma_{\langle R_{1x}\rangle} =$$

Control questions

1. What is the essence of the bridge method for measuring resistance? Which bridge leg is the rheostat of the bridge?
2. Write the Ohm's law for a homogenous branch of a circuit, a heterogeneous branch, a complete circuit.
3. Write the Ohm's law in a differential form.
4. Formulate the Kirchhoff’s rules. How to apply these rules for circuit calculations?
5. What is the formula of relation between the bridge legs when it is balanced? Derive it.
6. Can the Wheatstone bridge be used to measure small resistances? Analyze and justify the answer.

Laboratory work № 2-2

DEFINITION OF THE ELECTROMOTIVE FORCE USING THE METHOD OF COMPENSATION

Objective: to get acquainted with the compensation method for measuring the electromotive force (voltage).

Equipment: a Weston cell, a Leclanché cell (zinc–carbon battery) of unknown electromotive force (emf), a DC power supply UIP-2, a rheochord, a rheostat, a galvanometer, a switch.

Theoretical information

A work by Coulomb (electrostatic) force in carrying electric charge along a closed circuit equals zero. Therefore, the electrostatic field can not support continuous current in the circuit. The energy of current carriers is dissipated (the conductor is heating up) and its losses must be compensated. For this purpose, an arbitrary source of forces of non-electrostatic origin (a source of external forces) is used.

If E^{ex} is magnitude of the field of external forces, then the work in moving a charge q through the whole circuit is $A = q \oint E^{\mathrm{ex}} dl \neq 0$. So, the electromotive force (emf) is the work done by the external agent A_{ext} in carrying the electric charge q through the closed circuit (or between points 1 and 2 of the circuit) divided by the magnitude of that charge:

$$\varepsilon = \frac{A}{q} = \oint E^{ex} dl;$$

$$\varepsilon = \frac{A}{q} = \int_1^2 E^{ex} dl.$$

Emf is measured in the same units as the electric potential difference. In the international system of units (SI), this unit is volt (V).

Emf can be produced by the diffusion of ions in electrolytes, by the time-varying magnetic flux through a loop (electromagnetic induction), etc. The circuit branch, which includes the source of external forces, is called heterogeneous. For such a branch there is a generalized Ohm's law:

$$IR = \varphi_1 - \varphi_2 + \varepsilon_{12}, \quad (2.1)$$

here I is the current flowing in the direction from point 1 to point 2 (see Fig. 2.1); ε_{12} is the emf, R is the total resistance of the branch, $\varphi_1 - \varphi_2$ are electric potentials of points 1 and 2. Both I and i ε_{12} are algebraic quantities. If the emf contributes to the motion of positive charges in the chosen direction (B → A in Fig. 2.1), then $\varepsilon_{12} > 0$, if it counteracts, then $\varepsilon_{12} < 0$.

The sum of the potential difference at the terminals of the branch and the emf acting in this branch is called voltage:

$$U = \varphi_1 - \varphi_2 + \varepsilon_{12} \quad (2.1)$$

From formula (2.1) for the electromotive force we obtain:

1. If the circuit is open (I = 0), then

$$\varepsilon_{12} = \varphi_1 - \varphi_2, \quad (2.2)$$

that is, the source's emf is equal to the potential difference between two terminals of the unloaded source.

2. For the closed circuit $\varphi_1 = \varphi_2$ (points 1 and 2 coincide), we have

$$I = \frac{\varepsilon}{R} \quad (2.3)$$

Considering that the total resistance R of the circuit consists of the external resistance r_e, and the internal resistance r_i of the source of emf, that is $R = r_e + r_i$, the expression (2.3) can be rewritten as follows:

$$\mathcal{E} = Ir_e + Ir_i \Rightarrow \quad U = Ir_e = \varepsilon - Ir_i \quad (2.4)$$

It is clear that the voltage $U = Ir_e$ across the load resistance (external part of the circuit) is less than the emf by the value of the voltage drop across the internal resistance of the source of emf Ir_i. Therefore, it is impossible to measure the value of emf directly with a voltmeter. However, if the resistance of the voltmeter is very large comparing to the resistance of the circuit (it can be formally assumed that the resistance of the voltmeter goes to infinity, and in that case it is said that the voltmeter is ideal), then the current in the circгшe is zero (I = 0) and from (2.4) we obtain: U = Ɛ.

The essence of the present method used to determine the emf can be explained using a diagram depicted in Fig. 2.1. Two sources of emf Ɛ and $\mathcal{E}_1$ (Ɛ >$\mathcal{E}_1$) are connected opposite to each other. Resistances r_1 and r_1' are both variable, but there must be a condition

$r_1 + r_1' = \text{const.}$ (2.5)

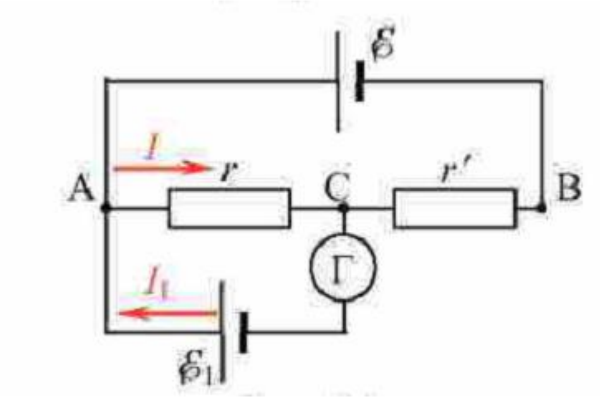


Figure 2.1.

By varying resistances r_1 and r_1', it is possible to adjust the potential difference between points A and C so that it is equal to the value of the emf $\mathcal{E}_1$. In that case, the current $I_1 = 0$. This means that

$$\mathcal{E}_1 = Ir_1 \tag{2.6}$$

It is clear now why the method is called the method of compensation: the absence of current in the circuit of the source $\mathcal{E}_1$ is due to the compensation of the emf of that source by the voltage drop across the branch AC.

If the source of emf $\mathcal{E}_1$ is replaced by a source whose emf is equal to $\mathcal{E}_2$, then we have to find new value r_2 of the resistance of the branch AC to compensate that emf:

$$\mathcal{E}_2 = Ir_2 \tag{2.7}$$

We must remember the condition (2.5): $r_1+r_1' = r_2+r_2' = r = const$

Dividing (2.6) by (2.7), we obtain

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{r_1}{r_2} \tag{2.8}$$

Proportion (2.8) is a working formula that is used to determine the unknown electromotive force if there is a source with known emf. Therefore, this method is also called the method of comparison.

Usually a Weston cell is used as the source with known emf. An anode of the Weston cell is an amalgam of cadmium with mercury while a cathode is of pure mercury. An electrolyte is a saturated solution of cadmium sulfate. Emf of the Weston cell is standard and equals $\varepsilon_N = 1{,}0183$ V.

If in formula (2.8) we replace $\mathcal{E}_2$ with $\mathcal{E}_N$, and $\mathcal{E}_2$ with $\mathcal{E}_x$, we obtain the working formula for determining the unknown electromotive force of the element $\mathcal{E}_N$:

$$\varepsilon_x = \varepsilon_N \cdot \frac{r_1}{r_2} \qquad (2.9)$$

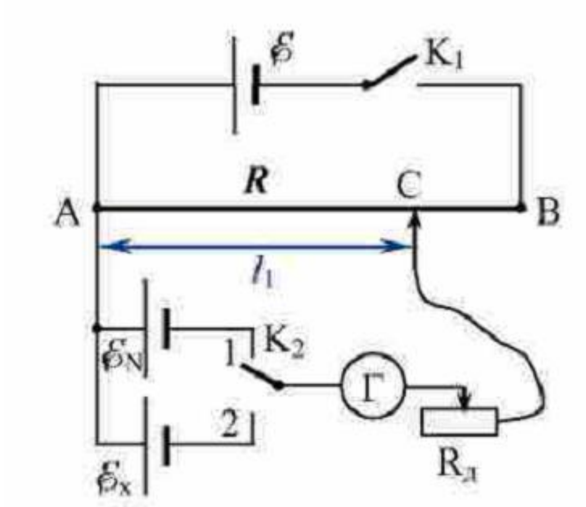


Figure 2.2.

In practice (see Fig. 2.2), section AB is a homogeneous string called rheochord. Resistance of the part AC is denoted with *R*. It is equivalent to r_1 (or r_2) from the discussion above, while the resistance of the part CB is equivalent to r_1' (r_2'), and the condition (2.5) is satisfied automatically.

Resistance *R* of the part of the rheochord AC, the voltage drop across which compensates the known emf $\mathcal{E}_N$, is proportional to the length l_1 of that part of the rheochord (in Figure 2.2 the switch K_2 is in position 1).

If we replace $\mathcal{E}_N$ with $\mathcal{E}_x$ (in Figure 2.2 the switch K_2 is in position 2), then the voltage drop across the part AC must change to compensate another, unknown emf $\mathcal{E}_x$. Resistance to the part AC and, accordingly, the voltage drop across that part, is varied by shifting the point C on the rheochord. Denote the new length of the part AC with l_1'. Then, the ratio of resistances $\frac{r_1}{r_2}$ in (2.13) can be replaced by the ratio of the corresponding lengths:

$$\frac{r_1}{r_2} = \frac{l_1}{l_1'}$$

Now the calculation formula is:

$$\varepsilon_x = \varepsilon_N \frac{l_1'}{l_1}. \quad (2.10)$$

Experimental setup

The electrical circuit shown in Fig. 2.2 is used to measure the emf.

A Leclanché cell (zinc–carbon battery) is used as the unknown emf. A DC power supply UIP-2 of output $\varepsilon = 3 - 9$ V is used.

A galvanometer Г is used to measure current through the branch of emf ε_N or ε_x. In order to restrict the current flowing through the galvanometer while the circuit is uncompensated, an additional resistor $R_Д$ is connected into the branch with galvanometer.

Procedure

1. Get acquainted with the electric circuit shown at the diagram in Fig. 2.2.
2. Set a switch K_2 into neutral position and set a slide-bar in the center of reochord C.
3. Set resistance $R_д$ to maximum value (~20 000 Ohm) and close the switch K_1.
4. Using the switch K_2, connect the galvanic element with known emf ε_N and by moving the slide-bar along the reochord achieve the null current in the circuit of the galvanometer Г (when the galvanometer reading reaches zero).
5. Turn off the emf ε_N and reduce the resistance $R_д$ to zero. Turn on the emf ε_N and, by moving the slide-bar of the reochord, find the more accurate position when the galvanometer's reading is readjusted to zero.

   Note that the circuit is to be completed for very short time only (just to take galvanometer’s reading) because current flow leads to heating of conductors and changes the resistances.
6. Measure the length of the reochord part AC = l_1, which provides absence of electric current, and write that value to the Table. 2.1. Repeat the experiment 3-5 times.
7. Perform the operations described in pp. 2-5, but connecting to the circuit (using the switch K_2) the unknown emf ε_x instead of ε_N. Write the values of the length of the reochord part AC = l_1' to the Table 2.1.

Table 2.1

Length of the reochord part, cm			Standard emf ε_N	Unknown emf ε_x	
n	l_1	l_1'		n	
1				1	
2			$\varepsilon_N = 1{,}0183$ V	2	
3				3	
4				4	
5				5	

8. Calculate the mean value of the unknown emf:

$$\langle \varepsilon_x \rangle = \frac{\sum_{i=1}^{5} \varepsilon_{xi}}{5} =$$

9. Calculate the standard error of measurement (see Theory of Errors, Lab No. 1-1).

10. Record the final result in the form

"desired value = mean value ε_x ± standard error":

Control questions

1. What is the emf of a source?
2. What is the role of the forces of non-electrostatic origin (external forces) in electric circuits?
3. What branch of an electric circuit is called heterogeneous? Write the Ohm's law for such a branch.
4. What is the essence of the compensation method?
5. Why does the emf ε have to be greater than ε_N and ε_x?
6. What are the benefits of the compensation method comparing to the direct emf measurement with a voltmeter?
7. What is the composition of a standard cell?
8. Why do the emfs ε, ε_N and ε_x should be connected with the terminals (poles) of the same sign?
9. What is the purpose of the resistance $R_Д$ in the circuit of the galvanometer?

Laboratory work № 2-3

DETERMINATION OF CAPACITANCE USING A BALLISTIC GALVANOMETER

Objective: to master the method of measuring capacitance of a capacitor using a ballistic galvanometer.

Equipment: a ballistic galvanometer, a DC power supply UIP-2, voltmeter, capacitors of unknown capacitance, capacitor of known capacitance, switches.

Theoretical information

Determination of a capacitor's capacitance is based on the definition:

$$C = \frac{q}{U} \qquad (3.1)$$

where *q* is the charge of the capacitor, *U* is the potential difference between the plates. From formula (3.1) we have $U = \frac{q}{C}$. If two capacitors with different capacitances (C_1, C_2) are charged to the same potential difference, then their charges (q_1, q_2) will be different, but $\frac{q_1}{C_1} = \frac{q_2}{C_2}$. Then we have:

$$C_2 = C_1 \frac{q_2}{q_1}. \qquad (3.2)$$

Thus, if capacitance of one of the capacitors is known, then capacitance of the second one can be calculated by formula (3.2), knowing the relation of charges of these capacitors (provided that the capacitors are charged to the same potential difference $U_1 = U_2 = U$.

In present work, comparison of charges of the capacitors is carried out using a ballistic method, in which a ballistic galvanometer is the recording device sensitive to currents of order of $(10^{-6} - 10^{-12})$ A.

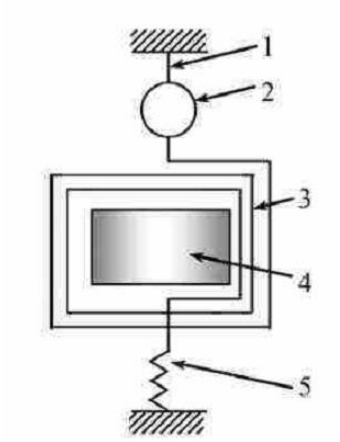


Figure 3.1.

Brief description of the ballistic galvanometer

A ballistic galvanometer is a type of sensitive galvanometers measuring the quantity of charge discharged through it. Unlike a current-measuring galvanometer, the moving part has a large moment of inertia, thus giving it a long oscillation period and making this device an integrator of current whose output signal is the time integral of its input signal.

The ballistic galvanometer is used to measure the amount of electricity (electric charge) that passes through the circuit in a time short comparing to the period of natural oscillations of the frame. Short-term currents take place in electric circuits during capacitor discharge, rapid changes in magnetic flux and so on. The ballistic galvanometer belongs to the devices of magnetoelectric system and differs from an ordinary galvanometer by the artificially increased moment of inertia of its moving part.

There are arrow and mirror galvanometers, the latter are more sensitive. In a mirror galvanometer (Fig. 3.1.), a movable frame 3 is suspended in magnetic field due to a stable magnet from a thin (with a thickness of a few microns) elastic thread. In order to amplify the magnetic field, an iron cylindrical core 4 is placed inside the frame.

Electric current flows to the frame through a pivot 1 and a thin metal thread 5 that draws down the moving system. A light mirror 2 is attached to the hanger near the frame. A ray of light from a light source is reflected from the mirror onto the scale of the galvanometer. The further is the scale from the mirror, the higher is the sensitivity of the device.

The increase in the moment of the inertia of the galvanometer can be achieved by attaching an additional load to the frame 3.

Motion of the frame of the ballistic galvanometer is, in general, influenced by three torques:

1) a torque exerted on the frame by the current passing through it; its maximum value is $iBSn$

2) a torque due to the torsion forces, $D\varphi$

3) a torque due to the resistance forces, $P\dot{\varphi}$

The fundamental equation of dynamics of rotational motion of the frame can be written as

$$I\ddot{\varphi} = -P\dot{\varphi} - D\varphi + iBSn, \qquad (3.3)$$

where I is the moment of inertia of the movable system of the galvanometer, B is magnitude of the magnetic field inside the air gap, S is the area of the frame, n is the number of wire turns in the frame, D is the torque of torsion forces per unit angle, i is the current flowing through the frame, P is the torque due to the resistance forces per unit angular velocity.

Let us use the formula (3.1) to establish relation between the magnitude of the charge q passing through the winding of the galvanometer frame and the first maximum deviation of the frame from the equilibrium position in the absence of resistance forces ($P = 0$).

Since during the passage of current the heavy frame of the galvanometer does not have time to go out of equilibrium, then the equation of motion of the frame during the time $\tau < T_0$, where T_0 is the period of natural oscillations of the frame (more precisely, of its first deviation), can be written approximately as follows:

$$I\ddot{\varphi} = iBSn, \qquad (3.4)$$

where

$$BSn\int_0^{\tau} i\,\mathrm{d}t = I\dot{\varphi}. \qquad (3.5)$$

In that case, the frame will acquire the kinetic energy $\frac{I\dot{\varphi}}{2}$. That energy is spent on torsion of the hanger by the angle φ. Since the torque due to the torsion forces equals

$D\varphi$, then when twisting the thread by the angle dφ the work by the torsion forces will be $\delta A = D\varphi \mathrm{d}\varphi$. When the frame is rotated by the angle φ_0, the work is

$$A = \int_0^{\varphi_0} D\varphi \mathrm{d}\varphi = \frac{D \cdot \varphi_0^2}{2} \qquad (3.6)$$

So,

$$\frac{D \cdot \varphi_0^2}{2} = \frac{I \cdot \dot{\varphi}^2}{2} \quad \Rightarrow \quad I \cdot \dot{\varphi}^2 = D \cdot \varphi_0^2. \qquad (3.7)$$

From the equations (3.5) and (3.7), we obtain:

$$I = \frac{B^2 \cdot n^2 \cdot S^2 \cdot q^2}{D \cdot \varphi_0^2}. \qquad (3.8)$$

The period of natural oscillations in the galvanometer's frame is equal to $T = 2\pi\sqrt{I / D}$, consequently $I = (T_0^2 \cdot D) / 4\pi^2$. Substituting this expression into (3.8), we obtain

$$\frac{T_0^2 \cdot D}{4\pi^2} = \frac{B^2 \cdot n^2 \cdot S^2 \cdot q^2}{D \cdot \varphi_0^2} \quad \Rightarrow \quad q = \frac{T_0 \cdot D}{2\pi BnS} \cdot \varphi_0 = \frac{T_0}{2\pi} \cdot c \cdot \varphi_0 = b \cdot \varphi_0, \qquad (3.9)$$

where $b = (cT_0) / 2\pi$. The value $c = D / (B \cdot n \cdot S)$ is called the dynamic constant, and b is the ballistic constant of the galvanometer.

It is shown above that under certain conditions $U_1 = U_2$ measurement of capacitance can be reduced to measurement of the ratio of charges of two capacitors, the capacity of one of which is considered to be known. From the formula (3.9) it follows that determination of the ratio of charges can be replaced by determination of maximum angular deviations φ_0 of the galvanometer frame, which are caused by current passing through it. So,

$$C_2 = C_1 \frac{\varphi_0^{(2)}}{\varphi_0^{(1)}}. \qquad (3.8)$$

Ratio of the maximum angular deviations of the galvanometer frame $\varphi_0^{(2)} / \varphi_0^{(1)}$ can be replaced by ratio of the maximum linear deviations of the reflected light ray (number of scale divisions) $n_0^{(1)} / n_0^{(2)}$ as far as during the experiment the distance from the mirror to the galvanometer scale remains constant. The angle φ of the frame rotation is related to the displacement n of the light stroke on the scale and to the distance l from the scale to the mirror, as we see from the Fig. 3.2.

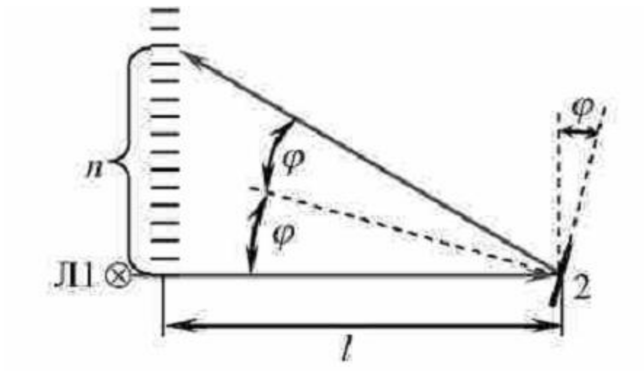


Figure 3.2.

It is obvious that the number of scale divisions traveled by the light stroke is directly proportional to the angle of deviation of the mirror $n \cdot \varphi$, and since the angle of deviation is proportional to the charge that has passed through the circuit, then the number of divisions is proportional to the charge: $n \cdot q$. If capacitors C_1, C_2 with charges q_1, q_2 are discharged through the galvanometer, then

$$q_1 = bn_1; \qquad q_2 = bn_2, \tag{3.11}$$

where *b* is the coefficient of proportionality.

Using the formula (3.2), we obtain

$$C_2 = C_1 \frac{n_2}{n_1}. \tag{3.12}$$

Experimental setup

Fig. 3.3 shows the circuit diagram used to determine capacitance of capacitors.

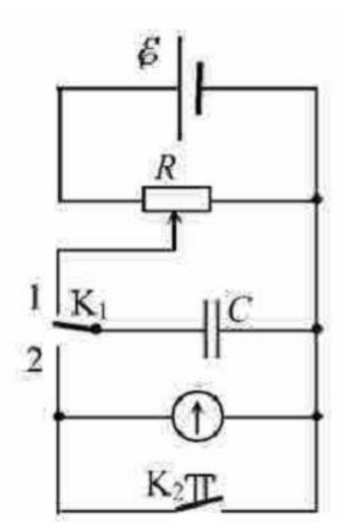


Figure 3.3.

A direct current from the power supply ε flows to the variable resistor *R* connected as a potentiometer (voltage divider). If the switch K_1 is in the position 1, then the voltage from the potentiometer is applied to the capacitor C and it is charging. The voltage can be varied by moving the slider of the variable resistor (rheostat). When the switch K_1 is in the position 2, then the capacitor is closed with the galvanometer.

A push-button switch K_2 is connected in parallel with the galvanometer. It completes the circuit of induction currents that arise in the galvanometer frame during its oscillations. Braking of the frame oscillations is reached as the result of magnetic field of the galvanometer's magnet acting on the induction currents (induction friction). So, the switch K_2 should be closed at the moment when the light stroke travels back through the zero scale division.

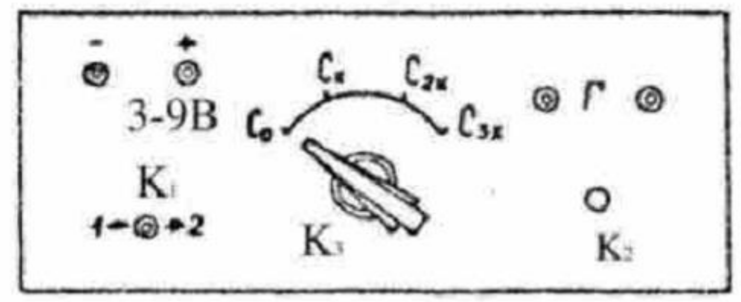


Figure 3.4.

The front panel of the experimental setup is shown in Fig. 3.4. It is supplied from the output "3 ... 9V" of the UIP-2. The ballistic galvanometer is connected to the terminal Г. The switch K allows to connect into the circuit consistently the reference capacitor C_0 and the unknown capacitors C_{1x}, C_{2x}, C_{3x}, C_{4x} to be measured.

Procedure

1. Turn on the power supply UIP-2 to a 220V network.
2. Using the potentiometer R (located on the UIP panel at the output "3 ... 9 V"), set the voltage U_1, (3V or 9V).
3. Set the switch K_3 into position C_0.
4. Set the switch K_1 into position 1 to charge the capacitor.
5. Set the switch K_1 into to position 2 to discharge the capacitor through the galvanometer. Measure the maximum deviation of the light stroke on the scale n_0^0 and write it to the Table 3.1. Repeat measurements according to pp. 4. 5 twice and determine the average value $\langle n_0^0 \rangle$.

Remark. Do not forget to use the switch button K_2 to brake oscillations of the galvanometer frame.

6. Using the switch K_3, connect the capacitor of unknown capacitance C_{1x} to the circuit instead of the reference capacitor.

7. Without changing the voltage U_1, perform measurements described in pp. 4-5 and calculate the average value $\langle n_0^1 \rangle$ ..

8. Perform operations described in pp. 3 - 7 for capacitors C_{2x}, C_{3x}, C_{4x}, writing the obtained data into tables 3.2, 3.3 and 3.4, respectively.

9. Set another voltage with the potentiometer (see p. 2), and repeat the measurements according to pp. 3-8. Record the results into tables 3.1, 3.2. 3.3 and 3.4.

Table 3.1

Capacitance C_{1x}					
U_1			U_2		
	n_0^0	n_0^1		n_0^0	n_0^1
1					
2					
$\langle n \rangle$					
$\langle C_{1x}(U_1) \rangle =$			$\langle C_{1x}(U_2) \rangle =$		

Table 3.2

Capacitance C_{2x}					
U_1			U_2		
	n_0^0	n_0^1		n_0^0	n_0^1
1					
2					
$\langle n \rangle$					
$\langle C_{2x}(U_1) \rangle =$			$\langle C_{2x}(U_2) \rangle =$		

Table 3.3

Capacitance C_{3x}					
U_1			U_2		
	n_0^0	n_0^1		n_0^0	n_0^1
1					
2					
$\langle n \rangle$					
$\langle C_{3x}(U_1) \rangle =$			$\langle C_{3x}(U_2) \rangle =$		

Table 3.4

Capacitance C_{4x}					
U_1			U_2		
	n_0^0	n_0^1		n_0^0	n_0^1
1					
2					
$\langle n \rangle$					
$\langle C_{4x}(U_1) \rangle =$			$\langle C_{4x}(U_2) \rangle =$		

Processing of the experimental results

Using the formula

$$\langle C_{ix}(U_i) \rangle = C_0 \cdot \frac{\langle n_0^i \rangle}{\langle n_0^0 \rangle},$$

determine the average values of unknown capacitances (for values from i = 1 to i = 4) for the two different charging voltages U_1, U_2. The final averaged experimentally measured values of capacitances should be calculated with the formula

$$\langle C_{ix} \rangle = \frac{1}{2}(\langle C_{ix}(U_1) \rangle + \langle C_{ix}(U_2) \rangle).$$

Determination of measurement error

1. Compare the measured average values of capacitance $\langle C_{1x} \rangle$ with the etalon capacitance shown on that capacitor $C_1(\text{cap})$ and calculate the relative error of measurement:

$$\varepsilon_1 = \frac{\Delta C}{\langle C_{1x} \rangle} \cdot 100\% = \frac{C_1(\text{cap}) - \langle C_{1x} \rangle}{\langle C_{1x} \rangle} \cdot 100\% =$$

2. Repeat calculations of the p.1 for $\langle C_{2x} \rangle, \langle C_{3x} \rangle, \langle C_{4x} \rangle$ to find errors $\varepsilon_2, \varepsilon_3, \varepsilon_4$.

3. Write the measurement errors

$\varepsilon_1 =$ ……………….; $\varepsilon_2 =$ ………………; $\varepsilon_3 =$ ………………; $\varepsilon_4 =$ ………………;

Control questions

1. What is an electric capacitance and what are its units?
2. What is the essence of the proposed method for measuring a capacitor's capacitance?
3. Derive a formula that confirms that the value of the first deviation of the ballistic galvanometer frame is proportional to the charge that has passed through it.
4. What is the construction of the ballistic galvanometer and what is its purpose?
5. What are the dynamic and ballistic constants of the galvanometer?
6. What is the purpose of the push-button switch K_2 in the circuit?
7. What is the working principle of the circuit diagram in Fig. 3.3?

Laboratory work № 2-5

STUDY OF THE ELECTROSTATIC FIELD

Objective: to get acquainted with one of the methods for studying characteristics of the electrostatic fields which is based on mathematical modeling using an example of the field with axial symmetry.

Equipment: a plate with electrodes, a microammeter, a power supply, a probe (electrode).

Theoretical information

When constructing electronic bulbs, electronic lenses and other devices, you need to know distribution of the electric field in space between electrodes of arbitrary shape. Theoretical calculation of such fields is possible only for the simplest field configurations, and in general such a calculation can not be performed. Therefore, electric fields of complex configuration are investigated experimentally, using the simulation method.

The main characteristics of the electrostatic field are the electric field vector and the electric potential. The electric field vector $\vec{E}$ is a force characteristic of the field which is numerically equal to the force exerted by the electric field on a single positive point charge q_0 located at some point in the field:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

Electrostatic fields are created by charged objects (or just electric charges). If the field is created by a point charge q, then the magnitude of the electric field vector at a distance r from that charge is determined by the formula:

$$E = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2} \ .$$

The energy characteristic of the field is the potential. For some types of fields, in particular, for the field due to a point charge, the potential is numerically equal to the work done by the forces of the field in moving a single positive point charge q_0 from a given point in the field to infinity without any acceleration:

$$\varphi = \frac{A}{q_0}.$$

Electric potential of the field due to the point charge depends on the distance r between the point where the potential is determined and the point where the source charge q is located:

$$\varphi = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r}.$$

A set of points in the field having the same potential forms an equipotential surface. Electric field lines are always perpendicular to the equipotential surfaces. Tangent to the electric field line at each point coincides with the vector of electric force acting on a test charge located at that point.

Method of mathematical modeling of the electrostatic fields

For the mathematical modeling of the electrostatic field, we use the fact that the electric field created by a stationary current in a medium with weak electric conductivity is always potential. That makes it possible to use the electric field of a stationary current in a medium with weak conductivity for modelling the electrostatic field of charged objects in vacuum.

The charged objects in the simulation are represented by electrodes, the shape of which corresponds to the shape of natural bodies, manufactured in a certain scale (usually magnified). Mutual arrangement of the electrodes must be the same as in the simulated device. The voltage supplied to the electrodes must be equal or proportional to the voltage across the electrodes of the simulated device. Under such conditions, electric field between the electrodes will have the same configuration as that of the simulated field, and will differ from it only by the numerical values of characteristics.

If the medium of weak electric conductivity is placed between the electrodes, the configuration of the electric field will change. However, under certain conditions, such changes can be avoided. What are these conditions?

The field distribution in space is given by the Maxwell equations for the electric field. The solution of these equations, which determines the law of distribution of the electric field, depends both on the form of the equations themselves and on the boundary conditions. We shall show that the form of the Maxwell equations will not change if we replace the non-conducting medium with the weakly conducting one.

As it is known, the electric current density j inside a conductor satisfies the equation of continuity:

$$\operatorname{div}\vec{j} = 0.$$

Using the differential form of the Ohm's law and taking into account that the specific electric conductivity of the medium is constant (σ = const), we obtain:

$$\operatorname{div}\vec{j} = \operatorname{div}(\sigma\vec{E}) = \sigma \operatorname{div}\vec{\mathrm{E}} = 0,$$

and

$$\operatorname{div}\vec{j} = 0. \tag{5.1}$$

In the absence of a time-varying magnetic field,

$$\operatorname{rot}\vec{E} = 0. \tag{5.2}$$

Thus, the electric field of the constant current satisfies equations (5.1) and (5.2). The electrostatic field in vacuum satisfies the same equations.

Equations (5.1) and (5.2) describing fields in the conducting medium and in vacuum have solutions that are dependent on the boundary conditions. Consider the boundary conditions on the electrodes and other interfaces between different media. If the electrical conductivity of the medium is small, then the current flowing through this medium is small as well. On the other hand, the electrical conductivity of the electrodes is large, so we can neglect the voltage drop across their volume and assume that the surfaces of the electrodes are equipotential. Therefore, the lines of current flow and electric field lines in the weakly conductive medium are perpendicular to the interface electrode/weakly conductive medium.

A similar configuration of the electric field lines is in a dielectric medium, for example, in vacuum. However, boundary conditions must be satisfied not only at the interface electrode/weakly conductive medium but also at other interfaces.

A conductive paper is used as the weakly conductive medium, and it is bordered by air and insulating substrate. Since at the boundaries between conductive paper, air and substrate the electric current can not pass perpendicular to the interface surface (from the non-conductive medium into the conductor), then inside the conducting medium the electric potential distribution is established in the way that there are no components of the electric field vector $\vec{E}$ perpendicular to the interface surface. Therefore, inside the conducting medium, the current flow lines and the electric field lines go along the interface with non-conductive medium. The interface does not introduce distortions into the shape of current lines and, consequently, into the shape of the field lines, if the contour of that interface corresponds to the contour of the current lines.

In the case when all these conditions are fulfilled, one can create a model of the field of electrostatic charges using the field in the weakly conductive medium. In such a simulation, the lines of current correspond to the field lines of the electrostatic field, and surfaces of identical voltage correspond to the equipotential surfaces. The advantage of such a simulation is that it is much easier to measures fields in the conducting medium than in non-conducting. That simplicity is due to the fact that in the conducting medium the electrical potential is measured instead of measuring the magnitude of electric field. For such measurements, probes (electrodes) that are introduced into the field are used.

The probe is a conductor which is well insulated along its entire length, except for its end. To measure the potential difference between two points of the field in the conducting medium, it is necessary to touch these points with probes connected to the voltmeter. This method can be used to draw lines of the same potential on the electroconductive paper. Current lines on the paper are perpendicular to the line of identical potential and correspond to the electric field lines inside of a cylindrical capacitor. That is of the same nature as distribution of electric potential of the electrostatic field due to an infinitely long, uniformly charged wire.

The field is modeled using an electrically conductive paper with two coaxial cylindrical electrodes tightly pressed to it. For such a model, the dependence of the magnitude and potential of the electric field on the distance r from the axis of the system can be calculated as follows. The current density $j(r)$ at the distance r from the axis can be found from the condition of continuity

$$j(r) = \frac{I}{2\pi r d},$$

where I is total current passing through the paper, d is the thickness of the paper.

Then, the magnitude of the electric field at the distance r is

$$E(r) = \frac{j(r)}{\sigma}, \text{ or } E(r) = \frac{I}{2\pi r d\sigma},$$

where σ is the electrical conductivity of the paper.

The experimental setup in the present work is assembled in a way that the potential difference between the external electrode, whose potential is assumed to be zero, and a given point in the field is measured during the experiment. Let's calculate theoretically that potential difference using the relationship between the electric potential and the electric field vector:

$$\vec{E} = -\text{qrad}\varphi.$$

In the polar coordinate system, this equality is rewritten as follows:

$$E(r) = -\frac{\mathrm{d}\varphi}{\mathrm{d}r} = -\frac{\mathrm{d}U(r)}{\mathrm{d}r},$$

then

$$U(r) = -\int_{r_e}^{r} E(r)\,\mathrm{d}r = \frac{I}{2\pi r d\sigma}\ln\frac{r_e}{r},$$

where r_e is the radius of the external electrode.

The obtained expressions for $U(r)$ and $E(r)$ can be rewritten in a more convenient form for research if instead of σ, I, d we introduce values r_e, r_{in}, U_0 which can be easily measured experimentally (r_{in} is the radius of the internal electrode, U_0 is the potential difference between the external and internal electrodes):

$$U_0 = U(r) = \frac{I}{2\pi r d\sigma}\ln\frac{r_e}{r_{in}}; \tag{5.3}$$

$$U(r) = \frac{U_0}{\ln\frac{r_e}{r_{in}}}\ln\frac{r_e}{r}; \tag{5.4}$$

$$E(r) = \frac{U_0}{\ln\frac{r_e}{r_{in}}}\frac{1}{r}. \tag{5.5}$$

The essence of the present work is to determine dependencies $U = U(r)$ and $E = E(r)$ experimentally and to compare the obtained results with the theoretical dependences given by formulas (5.4) and (5.5).

Description of the experimental setup

A diagram of the experimental setup for modelling the field vector and potential distribution for the electrostatic field with axial symmetry is schematically depicted in Fig. 5.1. Here 1 is the external electrode with the radius r_e; 2 is the conductive paper; 3 is the central electrode with the radius r_{in}; 4 is the probe (electrode); 5 is the micro-ammeter; $R_Д$ is the additional resistance. The electrical circuit is connected to a stabilized power supply U.

Such a circuit makes it possible to perform voltage measurements between the external electrode 1 and any point A of the conductive medium. The measuring device is a microammeter, which, due to the large additional resistance $R_Д$ ~ 100 kOhm, operates in voltmeter mode. Since the additional resistance $R_Д$ is large comparing to resistance of the region of the conductive paper between the point A and the external electrode, connection of the measuring circuit does not bring significant distortions into the current through that region and, consequently, into the voltage drop $U(r)$ between the point A and the external electrode. If we neglect the resistance of the contact zone between the probe and the conductive paper, as well as the internal resistance of the microammeter in comparison with the value $R_Д$, then the current flowing through the microammeter is:

$$I(r) = \frac{U(r)}{R_Д},$$

where r is the distance from the axis of the model to the point A.

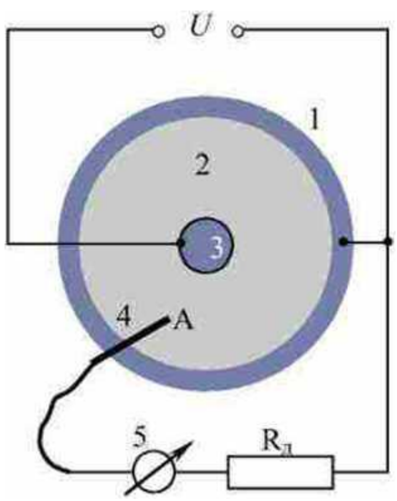


Figure5.1.

By measuring the current $I(r)$ passing through the microammeter, we find the voltage drop as:

$$U(r) = I(r)R_{Д}. \qquad (5.6)$$

For the more precise definition of $I(r)$, the measurements are repeated several times for the points located at different radii but at the same distance from the axis of the model. After that, we find the average value of the current:

$$< I(r) >= \frac{1}{n} \cdot \sum_{i=1}^{n} I_i(r). \qquad (5.7)$$

Then, for the voltage drop,

$$U(r) =< I(r) > R_{Д}.$$

The described experimental setup makes it possible to change r from 1 cm (internal electrode) to 8 cm (external electrode) with $\Delta r = 1$ cm increment. The value of $R_{Д}$ is indicated on the experimental setup.

Using the calculated values $U(r)$, we can find dependence of the magnitude of electric field at a given point of the conductive paper on the distance r. To do that, we use the relationship between $U(r)$ and $E(r)$:

$$U(r) = \frac{U(r) - U(r + \Delta r)}{\Delta r}.$$

In this work, the voltage drop $U(r)$ is measured at points located at distances $\Delta r = 0{,}01$ m from each other. The magnitude of electric field at those points can be calculated using the data for $U(r)$. However, such a calculation will not be accurate enough. We can determine the magnitude of electric field much more accurately at intermediate points with radii r = 1.5; 2.5; ...: 7.5 cm using the formula:

$$E(r + 0.5 \text{ cm}) = \frac{U(r) - U(r + 1 \text{ cm})}{0.01 \text{ cm}} \qquad (5.8)$$

in volts per meter (V/m) if $U(r)$ is given in volts.

Procedure

1. Using the description, get acquainted with the experimental setup. Prepare it for work. Turn on the power supply.
2. Press the measuring probe to the inner electrode and record the value of the

maximum current I_0 flowing through the device. Obviously, the maximum potential difference between the electrodes will be $U_0 = I_0 R_Д$ ($R_Д$ is the resistance of the additional resistor, which is indicated on the experimental setup).

3. Choose a ray (radius) on the conductive paper. Press the measuring probe consequently to the points on that radius at the distance of 1, 2, 3, … , 8 cm (Fig. 5.2) and read corresponding values of current $I_1(r)$ through the device from the microammeter. Write the data into the second column of Table 5.1.
   Remark. When measuring the current, the probe should be pressed not to the ray itself, but near it to the conductive paper. The probe should be pressed to the paper in order to provide good contact. You can notice the good electric contact by observing with the microammeter the current reaching its maximum value.
4. Repeat the measurements described in p.3 for the 2nd, 3rd and 4th rays. Fill the Table 5.1 with the obtained values of $I_1(r)$, $I_2(r)$, $I_3(r)$, $I_4(r)$.
5. Write the radii of the internal r_{in} and external r_e electrodes; write the value of the additional resistance $R_Д$.

Processing of experimental results:

1. Calculate the values of $U_0 = I_0 R_Д$.
2. Calculate: a) for each value of r (1, 2, 3 ... 8 cm) the average value of the current $I(r)$ according to the formula (5.7); b) the experimental value of the potential difference $U_{\exp}(r) = \langle I(r) \rangle R_Д$; c) the theoretical value of the potential difference $U_{theor}(r)$ according to the formula (5.4). Write the obtained values to the table. 5.1.
3. Using the formula (5.8) calculate the experimental values E_{exp} for r = 1.5; 2.5; 3.5; ...; 7.5 cm. Write the obtained results into the table. 5.2.
4. Using the formula (5.5) find the theoretical values E_{theor} for the same values of r as in p.3, and also write them to the table 5.2.
5. Using the data of the table 5.1, 5.2. build graphs of dependencies $E_{theor}(r)$ and $U_{theor}(r)$ and plot values of the experimental results on these graphs, highlighting the experimental points with circles.
6. For U and E estimate the average relative errors ε_U and ε_E of the experimental values

comparing to the theoretical ones.

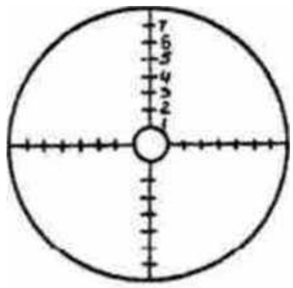


Figure 5.2.

Table 5.1

Radius r, cm	Current $I(r)$, μA, along the rays				Average current $\langle I(r) \rangle$, μA	$U(r)$, V	
	$I_1(r)$	$I_2(r)$	$I_3(r)$	$I_4(r)$		Experimental value $U_{\exp}(r) = \langle I(r) \rangle R_Д$	Theoretical value
1							
2							
3							
4							
5							
6							
7							
8							

Table 5.2

Radius r, cm	1.5	2.5	3.5	4.5	5.5	6.5	7.5
$E_{\exp}$, V/m							
E_{theor}, V/m							

Control questions

1. What is the electric field vector and potential of the electrostatic field?

2. What is the relationship between the electric field vector and potential at a given point in the electrostatic field?

3. What field is called potential? Prove the potential nature of the electrostatic field.

4. What is the essence of the method of simulating electrostatic fields with the help of currents in a weak-conductivity medium?

5. How to prove that the current lines are orthogonal to the equipotential surfaces?

6. Derive the formulas for $E(r)$ and $U(r)$ which are used in this work.

7. How are the measurements performed in this work?

8. How to calculate $E(r)$ with the measured values of $U(r)$?

Recommended literature:

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